HYDRODYNAMIC THEORY OF HEAT TRANSFER TO THROUGH-FLOW GAS AND LIQUID SUSPENSIONS

Z. R. Gorbis

Inzhenerno-Fizicheskii Zhurnal, Vol. 14, No. 4, pp. 626–632, 1968

UDC 536.242

A derivation of the heat transfer equation is based on an extension of the Reynolds analogy to turbulent gas and liquid dispersed suspensions with a low solids concentration at $\mathrm{Pr} \geq 1$ with allowance for the motion of the boundary layer.

In [1] a theoretical relation was obtained for determining the rate of heat exchange with through flows of the "gas suspension" type in cylindrical channels. In deriving this relation in the first approximation certain assumptions were made, the most important of which are the assumption of zero boundary layer velocity ($v' = 0$) and the assumption that $\mathrm{Pr}_f = 1$. For gas-dispersed flows with a low true volume concentration $\beta \simeq 0.03-3\%$, the experimental data indicate that these assumptions are quite acceptable.

In order to solve the problem for nonzero boundary layer velocity and $\mathrm{Pr}_f \geq 1$, it is necessary to depart from the simplified model and to take into account, together with the turbulent flow, the turbulent boundary layer, which consists of a transition layer and a laminar sublayer. We will examine the shear stresses and heat fluxes in the viscous sublayer and then in the intermediate layer and the turbulent core.

The laminar sublayer is characterized by the presence of considerable variation of the velocity and temperature of the flow components along the normal to the heating surface (along the y-axis). The longitudinal thermal diffusivity is small and in our case can be neglected. Moreover, for disperse flows with a low solids concentration it may be assumed that the rheological properties are almost Newtonian. Then the characteristics of the shear stress in the sublayer will be taken into account by the apparent viscosity μ_f in accordance with the approximation [2]

$$s_f = \mu_f \frac{\partial v_x}{\partial y} \cong \mu (1-\beta)^{-2.8} \frac{v'_x}{\delta_{ls}}. \tag{1}$$

Assuming that at the wall the velocity is zero and that the part of the velocity profile in the region of a laminar sublayer of thickness δ_{ls} is linear, we find the following expression for the change of flow velocity in the sublayer: $v'_x - v_w = v'_x$.

The specific heat flux density in this sublayer is determined by the heat conduction mechanism and, in accordance with Fourier's law,

$$q_l = -\lambda'_f \frac{\partial t_x}{\partial y} \cong -\lambda'_f \frac{t'_x - t_{\text{cT}}}{\delta_{lt}}. \tag{2}$$

Here, λ'_f is the apparent thermal conductivity in the sublayer of the disperse flow, which can be found from Maxwell's formula

$$\frac{\lambda_f}{\lambda} = \frac{2 + \lambda_{\text{par}}/\lambda - 2\beta (1 - \lambda_{\text{par}}/\lambda)}{2 + \lambda_{\text{par}}/\lambda + \beta (1 - \lambda_{\text{par}}/\lambda)}.$$

For gas suspension flows $\lambda_f \simeq \lambda$. In the general case the quantities δ_{ls} and δ_{lt} in (1) and (2) are not equal, though this is not usually noted. By analogy with a laminar boundary layer we assume that

$$\frac{\delta_{ls}}{\delta_{lt}} \cong \sqrt[3]{\frac{\nu_f}{a_f}} = \sqrt[3]{\mathrm{Pr}_f}. \tag{3}$$

It is easy to note that when $\mathrm{Pr}_f \cong 1$—gas suspension flows—$\delta_{ls} = \delta_{lt}$, while when $\mathrm{Pr}_f > 1$—liquid suspension flows—$\delta_{lt} < \delta_{ls}$.

Using (3), we express (2) in terms of (1):

$$q_l = s_f \frac{\lambda_f}{\mu} (1 - \beta)^{2.8} \frac{t'_x - t_w}{v'_x} \sqrt[3]{\mathrm{Pr}_f}. \tag{4}$$

Since $\mathrm{Pr}_f = \mu_f c_f / \lambda_f$,

$$q_l = s_f c_f \mathrm{Pr}_f^{-0.67} \frac{t'_x - t_w}{v'_x}. \tag{5}$$

In the intermediate buffer layer heat is transported by both conduction and turbulent mixing. In this case it is necessary to take into account both the molecular and the turbulent shear stresses. Usually, two different momentum and heat transport mechanisms are taken simultaneously into account in the form characteristic of the viscous sublayer. For our case we obtain

Comparison of Calculated and Experimental Data

B, m³/m³	Gas-graphite particles		Water-graphite particles	
	$\mathrm{Nu}_f/\mathrm{Nu}$ acc. to (15)	$\mathrm{Nu}_f/\mathrm{Nu}$ acc. to [1]	$\mathrm{Nu}_f/\mathrm{Nu}$ acc. to (13)	$\mathrm{Nu}_f/\mathrm{Nu}$ acc. to [5]
0.005	2.0	2.0	1.0	1.0
0.01	3.0	2.97	0.985	0.990
0.03	7.0	6.8	0.95	0.94
0.05	—	—	0.94	0.91
0.10	—	—	0.895	0.857
0.20	—	—	0.765	0.692

$$q_{\text{b}} = (\lambda_{\text{f}} + \rho_{\text{f}}\, c_{\text{f}}\, \varepsilon_a) \frac{\partial t_x}{\partial y}, \tag{6}$$

$$s_{\text{b}} = (\mu_{\text{f}} + \rho_{\text{f}} \varepsilon_{\text{v}}) \frac{\partial v_x}{\partial y}. \tag{7}$$

Here, ε_a and ε_ν are the turbulent analogs of the thermal diffusivity and kinematic viscosity for the disperse flow, taking into account the contribution of the turbulence of the flow components to the total transport across the buffer layer. As distinct from a and ν the molar coefficients ε_a and ε_ν are not physical parameters and depend on various characteristics of the disperse flow ($\beta, d_{\text{par}}/D, \text{Re}, \ldots$). The molar coefficients are quantities hard to determine for homogeneous and particularly disperse flows.

The turbulent flow outside the buffer layer is characterized by fluctuational heat and momentum transfer

The turbulent flow outside the buffer layer is characterized by fluctuational heat and momentum transfer so intense that the effect of molecular transport can be neglected. Then (6) and (7) give

$$q = s c_{\text{f}} \frac{\varepsilon_a}{\varepsilon_\nu} \frac{\partial t_x}{\partial y} \bigg/ \frac{\partial v_x}{\partial y}. \tag{8}$$

For homogeneous flows it is often assumed that $\varepsilon_a/\varepsilon_\nu = 1$. For disperse flows such an approach is scarcely applicable. In view of the difficulty of using (8), we will determine the heat transfer to the outer edge of the laminar sublayer from the heat balance expression, assuming $\varphi_{\text{t}}' \cong \varphi_{\text{t}} = t_{\text{par}}/t$:

$$q = \frac{G'}{F} c_p (t_x - t_x') + \frac{G'_{\text{par}}}{F} c_{\text{par}} (t_{x\text{par}} - t'_{x\text{par}}) = $$
$$= \frac{\varphi_t}{F} (t_x - t_x')(G' c_p + G'_{\text{par}} c_{\text{par}}). \tag{9}$$

The corresponding shear stresses are determined in accordance with the law of conservation of momentum and the additivity principle, assuming $\varphi_{\text{v}}' \cong \varphi_{\text{v}} = v_{\text{par}}/v$:

$$s_{\text{f}} = s + s_{\text{par}} = \frac{G'}{F} (v_x - v_x') + \frac{G'_{\text{par}}}{F} (v_{x\text{par}} - v'_{x\text{par}}) = $$
$$= \frac{\varphi_v}{F} (v_x - v_x')(G' + G'_{\text{par}}). \tag{10}$$

Combining (9) and (10), we obtain

$$q = s \frac{c_p (t_x - t_x')}{v_x - v_x'} \left(1 + \frac{s_{\text{par}}}{s} \frac{c_{\text{par}}}{c_p} \frac{\varphi_t}{\varphi_v} \right) = $$
$$= s c_p \frac{t_x - t_x'}{v_x - v_x'} A. \tag{11}$$

Considering the stationary thermal regime, i. e., $dt/d\tau = dt_{\text{par}}/d\tau_{\text{par}} = 0$, we obtain $q_1 = q$. Then from (5) and (11) we find that

$$t_x' - t_{\text{w}} = \frac{q}{s_{\text{f}} c_{\text{f}}} \Pr_{\text{f}}^{0.67} \frac{v_x'}{v_x} v_x,$$
$$t_x - t_x' = \frac{q}{s c_p A} \left(1 - \frac{v_x'}{v_x} \right) v_x,$$

whence

$$\frac{q}{t_x - t_{\text{w}}} = \frac{s c_p}{v_x} \frac{A}{1 + \dfrac{v_x'}{v_x} \left[\Pr_{\text{f}}^{0.67} \dfrac{c_p}{c_{\text{f}}} A - 1 \right]}, \tag{12}$$

where

$$A = 1 + \frac{s_{\text{par}}}{s} \frac{c_{\text{par}}}{c_p} \frac{\varphi_t}{\varphi_v} = \frac{\varepsilon_a}{\varepsilon_\nu} > 1.$$

Considering that as $\varphi_{\text{t}} \to 1$ $\bar{t}_{\text{f}}$ and the heat transfer coefficient of the disperse flow α_{f} are given by

$$\bar{t}_{\text{f}} = \frac{1 + (\beta \rho_{\text{par}} c_{\text{par}} \varphi_t / \rho\, c_p)}{1 + (\beta \rho_{\text{par}} c_{\text{par}} / \rho\, c_p)} \bar{t} \cong \bar{t} \simeq t_x,$$
$$\alpha_{\text{f}} \equiv \frac{q}{\bar{t}_{\text{f}} - \bar{t}_{\text{w}}} \simeq \frac{q}{t_{\text{x}} - \bar{t}_{\text{w}}}$$

and that $s_{\text{f}}/s = \xi_{\text{f}}/\xi$; $s_{\text{par}}/s = (\xi_{\text{f}} - \xi)/\xi$ [1] and noting that for a homogeneous medium, in accordance with the Prandtl solution, which improves the Reynolds formula by taking into account heat transfer across the boundary layer

$$\alpha = \frac{s c_p}{v} \frac{1}{1 + \dfrac{v'}{v} (\Pr - 1)} = \frac{s c_p}{v} E,$$

instead of (12) we obtain the following, more general than in [1], equation of the hydrodynamic theory of heat transfer for a disperse flow:

$$\frac{\alpha_{\text{f}}}{\alpha} \cong \left[1 + \left(\frac{\xi_{\text{f}}}{\xi} - 1 \right) \frac{c_{\text{par}}}{c_p} \frac{\varphi_t}{\varphi_v} \right] E_{\text{f}}; \quad \frac{\text{Nu}_{\text{f}}}{\text{Nu}} = $$
$$= \frac{\alpha_{\text{f}}}{\alpha} \frac{\lambda}{\lambda_{\text{f}}}, \tag{13}$$

where

$$E_{\text{f}} = \left\{ 1 + \frac{v'}{v} (\Pr - 1) \right\} \left\{ 1 + \frac{v'}{v} \times \right.$$
$$\left. \times \left[\Pr_{\text{f}}^{0.67} \frac{c_p}{c_{\text{f}}} \frac{1 + (\xi_{\text{f}}/\xi - 1) \dfrac{c_{\text{par}}}{c_p} \varphi_t / \varphi_v}{\xi_{\text{f}}/\xi} - 1 \right] \right\}^{-1}. \tag{14}$$

Comparing this expression with that obtained in [1]

$$\frac{\alpha_{\text{f}}}{\alpha} \cong 1 + \left(\frac{\xi_{\text{f}}}{\xi} - 1 \right) \frac{c_{\text{par}}}{c_p} \frac{\varphi_t}{\varphi_v}, \tag{13'}$$

we note that they differ with respect to E_{f}, which takes into account the effect of $\Pr > 1$ and the motion of the laminar sublayer.

We will consider certain extreme and particular cases:

a) in the absence of discrete particles $\beta = 0$; $\xi_{\text{f}} = \xi$; $\xi_{\text{par}} = 0$, $\Pr_{\text{f}} = \Pr$; $c_{\text{f}} = c_p$; then from (14) $E_{\text{f}} = 1$, and from (13) $\alpha_{\text{f}}/\alpha = 1$ and $\alpha_{\text{f}} = \alpha$;

b) when $\Pr = 1$ $E = 1$, but $E_{\text{f}} < 1$;

c) when $v_{\text{x}}' = 0$ (stationary sublayer) $E_{\text{f}} = 1$, and (13) coincides with (13').

In deriving (13) no constraints were imposed on the Prandtl number. Therefore in general form the solution obtained is suitable for analyzing both gas and liquid disperse flows with a turbulent transporting medium at low volume concentrations. The latter constraint is associated with the effect of increased concentration on the flow structure and properties (intensification of the non-Newtonian properties of the system, reduced degree of freedom of behavior of the discrete particles, significant change in the rate of intercomponent heat transfer, redistribution of the thermal resistances of the characteristic flow layers, etc.) [1].

These factors essentially determine the limiting critical concentration beyond which the expressions obtained are invalid. For a gas suspension we have estimated these concentration values at $\beta_{cr} \cong 3\%$, which corresponds to a flow-rate concentration $\mu_{cr} = 45-50$ (kg/hr)/(kg/hr). In this case $\lambda_f \cong \lambda$; $c_f \cong c_p$; $Pr_f = Pr/(1-\beta)^{2.8}$; $\xi_f/\xi = 1 + k\mu = 1 + k(\rho_{par}/\rho)\,\beta\varphi_v$; $k = \varphi(Re, Re_{par}, \mu, D/d_{par} \ldots)$. Then for a gas suspension general expression (13) takes the form

$$\frac{\alpha_f}{\alpha} = \frac{Nu_f}{Nu} \cong \left[1 + k\frac{c_{par}}{c_p}\frac{\rho_{par}}{\rho}\beta\varphi_t\right]E_f, \tag{15}$$

$$E_f \cong \frac{1}{1 + \frac{v'}{v}\left[(1-\beta)^{-1.9}\frac{\xi}{\xi_f}\left(1 + \frac{\xi_{par}}{\xi}\frac{c_{par}\varphi_{par}}{c_p\ \varphi_v}\right) - 1\right]}. \tag{16}$$

Clearly, for liquid flows β_{cr} considerably exceeds 3% and approaches 20%. Here, the relative hydrodynamic drag characteristic can be determined from the formula based on the data of [3]

$$\frac{\xi_f}{\xi} = \left(\frac{Re_f}{Re}\right)^{0.25} = $$
$$= \left(1 - \beta + \frac{\rho_{par}}{\rho}\beta\right)^{-0.25}(1-\beta)^{-0.7}.$$

In any case all the quantities entering into (13) and (14) are physical characteristics either of the flow components (c_p, c_{par}, ρ, ρ_{par}, ν, λ, λ_{par}, ...) or of the entire system (β, c_f, ξ_f, φ_t, φ_v, ...), which must be known or estimated in advance. Obviously, the expressions giving in relative form the relation between the heat transfer rate and the hydrodynamic drag of the disperse flow can be used to analyze the effect of the various factors on the heat transfer characteristics or for a direct, though undoubtedly approximate, heat transfer calculation only if laws of the type ξ_f/ξ are known. As for the significance of the other characteristics of the disperse system, it must be emphasized that it is important to estimate the direction of the influence of growing concentration on the variation of the relative heat transfer rate Nu_f/Nu as compared with unity; in this case β exerts an influence not only through ξ_f/ξ but also through Pr_f and c_p/c_f.

The determination of v'/v presents difficulties, especially for media with $Pr > 1$ (liquids). For gases the choice of a method of estimating the velocity simplex cannot introduce a significant error, since, in accordance with (16), the complex E_f is only a few percent less than unity and in first approximation need not be taken into account at all. For homogeneous flows, on the basis of experiments Prandtl has proposed the approximation $v'/v \simeq 0.3$. L. S. Leibenzon has theoretically calculated this ratio for a liquid pipe flow with a parabolic variation of velocity in the laminar boundary layer and obtained $v'/v = 0.33$. There have been other recommendations, for example, $v'/v = 1.74\ Re^{-1/8}$ or $v'/v = 1.5\ Re^{-1/8}Pr^{-1/6}$.

The slip factors of the components with respect to velocity and temperature (φ_v and φ_t) have not been formally introduced. Their appearance is predetermined by the heterogeneity of the flow system. In the general case the value of these coefficients is determined as the sum of the hydrodynamic and thermal intercomponent interactions. To estimate φ_t it is necessary to determine the large-scale fluctuation time τ_t required for the establishment of intercomponent temperature equilibrium. In accordance with [1], if $\tau_{par} \geq 0.7d^2_{par}/\alpha_{par}$, then φ_t may be taken equal to unity, since the transverse fluctuation time is sufficient for there to be no intercomponent temperature drop and for $t = t_{par}$ at any point of the flow cross section. In accordance with [4], the particle transverse fluctuation period

$$T_{par} \cong \frac{D}{v_{par.\,flu}} = \frac{D}{v_{flu}}\sqrt{\frac{1 + tg^2\varphi}{\left(\frac{1.5\rho}{\rho_{par} + \rho}\right)^2 + tg^2\varphi}}.$$

In the first approximation the velocity slip factor can be estimated for the stabilized flow region:

$$\bar{\varphi}_v = \frac{\bar{v}_{par}}{\bar{v}} = 1 - \frac{v_w}{\bar{v}}.$$

Obviously, for small or light particles $v_w \ll v$ and $\varphi_v \cong 1$.

In the more general case, with allowance for particle acceleration, but assuming that the flow over the particles is self-similar (for irregular particles with a nonsphericity factor of 1.15–1.5 at $Re_{par} > 100-200$), in accordance with [1]

$$\varphi_v = 1 - \frac{v_w}{v}\ \mathrm{cth}\,\gamma,$$
$$\gamma = \frac{g\tau}{v_w} + 0.5\ \ln\frac{v_{rel} + v_w}{v_{rel} - v_w}.$$

The fluctuation velocity slip factor for any moment of time [4]

$$\varphi_{v_{flu}} = \frac{\bar{v}_{par.\,flu}}{\bar{v}_{flu}} = \sqrt{\frac{\left(\frac{1.5\rho}{\rho_{par} + 0.5\rho}\right)^2 + tg^2\varphi}{1 + tg^2\varphi}}.$$

$$tg\,\varphi = \frac{gD}{4v\,v_w\,\mathrm{cth}\,\gamma}\ln(Re - 0.9)\frac{\rho_{par} - \rho}{\rho_{par} + 0.5\rho}.$$

Expressions (13) and (15) were compared with experimental data [1, 5] for gas and liquid suspensions of graphite particles at $Re \equiv 3 \cdot 10^4$, $v'/v = 0.33$, $\bar{t} = 100°$ C, and $Re_{par} = 8$ (see table).

In addition to the satisfactory agreement between the experimental and theoretical data, the table reveals that an increase in solids concentration has differently directed effects on Nu_f/Nu. Thus, there is a significant intensification of heat transfer for gas-graphite suspensions and a certain reduction for water-graphite suspensions (for other aqueons suspensions Nu_f/Nu may also be greater than unity, but only very slightly—by a few percent). This, at first glance, paradoxical result was first examined in [1], but a fuller explanation can be obtained on the basis of an analysis of (13) from the standpoint of the hydrodynamic theory of heat transfer.

NOTATION

s denotes shear stresses; μ is the coefficient of dynamic viscosity; v is the velocity; β is the true volume concentration; λ is the thermal conductivity of the liquid (gas); t is the temperature; δ_{ls} and δ_{lt} are the thicknesses of hydrodynamic and thermal boundary layers; c_p is the mass specific heat of liquid at constant pressure; ρ is the density; Re, Pr, Nu are the Reynolds, Prandtl, and Nusselt numbers of liquid; d_{par} and D denote the diameter of particles and channel; ξ is the coefficient of friction; φ_v and φ_t are the nonuniformity factors, "slip" of flow components with respect to velocity and temperature; k is the analog of Gasterstädt coefficient; G' and G'_{par} are the transverse mass flow rate of continuous and discrete flow components caused by large-scale turbulent fluctuations; F is the channel heating surface; t_x, t_{xpar}, v_x, and v_{xpar} are the temperatures and velocities of flow components in the turbulent core; v_{cr} is the critical particle velocity; τ is the time of motion; v_{rel} is the relative initial velocity; α is the coefficient of heat transfer between flow and heating surface. Subscripts: par) solid particles; f) disperse flow; w) wall; flu) fluctuating quantities; prime) quantities at edge of laminar sublayer.

REFERENCES

1. Z. R. Gorbis, Heat Transfer of Disperse Through Flows [in Russian], Energiya, 1964.
2. Yu. P. Gupalo, IFZh, no. 2, 1962.
3. V. I. Kofanov, Energomashinostroenie, no. 1, 1964.
4. Z. R. Gorbis and E. F. Spokoinyi, Proc. 1st All-Union Interuniversity Conf. on Processes in Disperse Through Flows [in Russian], Odessa, 1967.
5. A. P. Miller and R. W. Moulton, Trans. Engin. Univ. Wash., 8, no. 2, 1959.

20 September 1967

Lomonosov Technological
Institute, Odessa